Summer Packet

for Rising Algebra 1 and Algebra 1 Honors Students

Rachel Carson Middle School

Dear Rachel Carson Algebra Student,

We hope that you will enjoy your summer vacation to the fullest. We look forward to working with you next year in Algebra. As you enter your Algebra class you will want your math skills to be as up to date as possible. You may need to brush up on some basic topics. The Algebra teachers have prepared this booklet of problems to prepare you for your Algebra course. You will be expected to know the fundamentals of algebra that are reviewed in this booklet.

The problems will be reviewed briefly and assessed during your first week of school in September. You will have the opportunity to ask questions and receive help on this material before the assessment.

Please do each problem and show all of your work in the booklet. Attach any additional work you may have to this packet.

If you or your parents have questions concerning this packet, please direct them to us at Rachel Carson Middle School.

Sincerely,

The Rachel Carson Algebra teachers

1: Order of Operations

Steps:

1. Simplify expressions within grouping symbols, such as parentheses, brackets, and fraction lines.

2. Simplify exponents and roots.

3. Do multiplication and division in order from left to right.

4. Do addition and subtraction in order from left to right.

Example: Evaluate	$5+12\div 2\cdot 3$
Divide 12 by 2	$5+6\cdot 3$
Multiply 6 by 3	$5+18$
Add	23

Practice Problems:

1. $5+6\div 3\cdot 7$

2. $9+13-3\cdot 7$

3. $\frac{3(2+1)}{9}+\frac{7(4+2)}{5-4}$

4. $2^2+(7-3)\div 2+(7-2)^2$

5. When simplifying, explain why $3^2+6\div 3\neq 5$.

2: Adding and Subtracting Integers

Notes:
- A number with no sign is considered a positive integer.
- An addition or subtraction sign also acts as the sign of the following number.

Rules:
- When adding or subtracting two numbers with the **same signs**, add the values and keep the sign.

  Examples: $4+2=6$ $-4-8=-12$

- When adding or subtracting two numbers with **different signs**, subtract the values and keep the sign of the larger number.

  Examples: $8-2=6$ $5-9=-4$
  $-6+2=-4$ $-5+8=3$

Practice Problems:

6. $-9+12$

7. $5-14$

8. $-11-13$

9. $-7+7$

10. $-5-5$

11. $5-3-9$

12. $13+4-5$

3: Multiplying and Dividing Integers

Rules:

- When two numbers have the **same signs**, multiply or divide the numbers. The answer will be positive.
  Examples: $3 \cdot 4 = 12$ $-14 \div -2 = 7$

- When two numbers have **different signs**, multiply or divide the numbers. The answer will be negative.
  Examples: $15 \div -3 = -5$ $6 \cdot -5 = -30$

Practice Problems:

13. $\dfrac{-12}{-3}$

14. $20 \div -5$

15. $-5(3)$

16. $4 \cdot (-3)$

17. $-4 \cdot (-6)$

18. $\dfrac{-3+9}{2}$

19. Find the average of $-8, -5, 4, 13, -12$, and -4.

4: Mixed Review

Simplify each expression.

20 $-3+5\cdot 2$

21. $-4\div 2+12/6$

22. $\dfrac{5-15}{-7+2}-3^2$

23. $(3+2)^2-7\cdot 4$

Write and simplify an expression to answer the following questions.

24. Tom's bank account is overdrawn by $24. He deposits a check for $50. Find Tom's new account balance.

25. A plumber is paid $50 to make a house call. She charges $80 per hour of work once she is there. How much does she earn for a 4-hour job?

Write a realistic problem that could be answered with the following expression.

26. $4\cdot 3+5\cdot 7$

5: Fraction Review

Notes:
- When adding and subtracting fractions you must have a common denominator.
- To divide fractions you write the problem as a multiplication by the reciprocal of the divisor.

Practice Problems:

Add or subtract as indicated. Write your answer in lowest form.

27. $\frac{2}{5}+\frac{1}{5}$

28. $\frac{5}{7}-\frac{2}{7}$

29. $\frac{1}{3}+\frac{1}{2}$

30. $\frac{2}{3}+\frac{3}{5}$

31. $\frac{3}{4}-\frac{1}{3}$

32. $\frac{1}{2}-\frac{1}{6}$

33. $5\frac{1}{4}+\frac{1}{2}$

34. $3\frac{1}{5}-\frac{2}{3}$

35. $3-\frac{1}{7}$

36. $\frac{5}{6}+\frac{7}{12}$

37. $\frac{13}{16}+\frac{5}{8}$

38. $\frac{19}{24}-\frac{5}{8}$

39. $3\frac{1}{4}-1\frac{7}{8}$

40. $5\frac{1}{6}-2\frac{3}{4}$

41. $6\frac{7}{8}+3\frac{5}{6}$

42. $4\frac{5}{12}-3\frac{13}{16}$

Multiply or divide as indicated. Write your answers in lowest form.

43. $\frac{1}{5}\times\frac{3}{4}$

44. $\frac{5}{6}\times\frac{3}{4}$

45. $\frac{5}{16}\div\frac{3}{4}$

46. $1\frac{5}{6}\div\frac{1}{3}$

47. $2\frac{1}{2}\cdot\frac{2}{3}$

48. $\frac{2}{5}\cdot 45$

49. $2\frac{3}{4}\cdot 16$

50. $2\frac{7}{8}\div\frac{1}{4}$

51. $3\frac{1}{6}\div 4\frac{5}{6}$

52. $6\frac{2}{3}\div 2$

53. $14\div\frac{7}{8}$

54. $27\div\frac{9}{16}$

55. $33\div 1\frac{3}{8}$

56. $58\div 3\frac{5}{8}$

57. $16\div\frac{2}{3}$

58. $2\frac{3}{5}\div\frac{13}{5}$

6: Distribution and the Combining of Like Terms

Distributive Property:

	Examples:
$a(b+c)=ab+ac$	$3(x+5)=3x+15$
$(b+c)a=ba+ca$	$(2+y)x=2x+yx$
$a(b-c)=ab-ac$	$4(t-r)=4t-4r$
$(b-c)a=ba-ca$	$(x-2y)6=6x-12y$

Note:
There must be addition or subtraction inside the parenthesis. There may also be more than two terms in the parenthesis.

Examples: $3(2x-4+w^3)=6x-12+3w^3$

$-4(3a+7b-5c-cd)=-12a-28b+20c+4cd$

Non-example: $2(5x)(3)=30x$

The 2 is not distributed since there is only multiplication involved.

Combining Like Terms:

Like terms have exactly the same variables raised to the same power. Like terms can be combined by addition or subtraction.

Examples: $3x+9x-2x=10x$

$6x^2-4x+5x^2=11x^2-4x$

$7y+3(2-6y)=7y+6-18y=-11y+6$

Practice Problems:

Simplify each expression.

59. $-7(3x^2+5x-9)$

60. $14x^2-6x+8-6x^2+x-11$

61. $4a-(3x-2)$

62. $7(3-2x)+8-(4x-9)$

63. $3x-(2x+4)+3(2x+5)+4x-1$

64. $(14+7a)\frac{1}{7}$

7: Evaluating Expressions

Rules:

- Replace each letter with its given value written in parenthesis.
- Use the correct order of operations to simplify the resulting expression.

Example: Evaluate $x - y$ if $x = 9$ and $y = 3$.

Substitute $(9) - (3)$

Simplify 6

Note:

Remember that $3x$ means "3 multiplied by x." When evaluating, be sure to put a symbol or parenthesis to indicate multiplication.

Example: Evaluate $2a - 3b$ if $a = -4$ and $b = -1$.

Substitute $2(-4) - 3(-1)$

Simplify $-8 + 3$

-5

Practice Problems:

Evaluate each of the following expressions for the given values of the variables.

$a = 5$	$b = -2$	$c = 0$
$d = -4$	$e = 1$	$f = 3$

65. $3a + 4e$

66. $|a - f| + |f - a|$

67. $2(d - 2)$

68. $\dfrac{2b - 4d}{3}$

69. $b^2 - 2df$

70. $5a + 6d - \dfrac{bf}{6}$

8: One-Step Equations with Addition or Subtraction

Notes:
- In an equation, a letter (variable) represents an unknown number.
- The variable can be on either side of the equal sign.
- An equation is completely solved when the variable is alone on one side of the equal sign.

Rules:
- Use the opposite operation to move the number away from the variable.
- Write the opposite operation directly beneath the original operation and directly beneath the number on the other side of the equation.
- Add down both sides of the equal sign and simplify.

Example:
$$x - 7 = -4$$
$$\underline{+7} = \underline{+7}$$
$$x + 0 = 3$$
$$x = 3$$

Practice Problems:

71. $g + 3 = 8$

72. $5 = x - 8$

73. $y + 6 = -3$

74. $-3 = -2 + f$

75. $x + \frac{7}{8} = -\frac{3}{4}$

76. $x - \frac{1}{4} = -7$

77. $x + 4.3 = -8.1$

78. $x - 9.94 = 2$

9: One-Step Equations with Multiplication or Division

Rules:

- If a variable is multiplied by a number, divide both sides of the equation by that number to undo. Example:

$$4x = -20$$
$$\frac{4x}{4} = \frac{-20}{4}$$
$$x = -5$$

- If a variable is divided by a number, multiply both sides of the equation by that number to undo. Example:

$$\frac{y}{3} = 9$$
$$3\left(\frac{y}{3}\right) = 3(9)$$
$$y = 27$$

Practice Problems:

79. $-3x = -12$

80. $14 = -2m$

81. $\frac{y}{8} = 2$

82. $\frac{x}{-3} = 6$

83. $-10 = \frac{w}{5}$

84. $7x = 7$

85. $-\frac{1}{9}x = -7$

86. $-\frac{6}{7}d = 36$

87. $\frac{3y}{7} = 24$

88. $-\frac{5}{7}k = 35$

10: Solving Multi-Step Equations

Strategy:
The goal is to get the variable alone on one side of the equation by following these steps.

- Simplify each side of the equation. Do any possible distribution and combining of like terms.
- Undo any addition or subtraction.
- Undo any multiplication or division.

Example:

$4(y+5)-y=12$

$4y+20-y=12$ - Distribute the 4

$3y+20=12$ - Combine like terms

$\underline{-20}=\underline{-20}$ - Undo adding 20 by subtracting 20 from both sides

$3y=-8$

$\frac{3y}{3}=\frac{-8}{3}$ - Undo multiplication by dividing both sides of the equation by 3

$y=-\frac{8}{3}$

Practice Problems:

89. $0.3p+1=2.2$

90. $21=3(2-a)$

91. $3(w+5)+w=5$

92. $5x-2(x+1)=10$

93. $8x+3(2-3x)=28$

94. $\frac{1}{4}(8-10x)+\frac{1}{2}x=5$

95. The length of a rectangle is $3x-5$. Its width is x. Find the length and width if the perimeter is 38.

11: Prime Factoring of Monomials

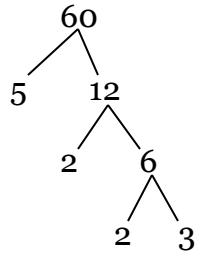
Notes:

- A "monomial" is an expression without any addition or subtraction in it. These are monomials: -5, $12x$, and $3x^2$.
- To "factor" means to rewrite an expression as a multiplication problem. For example, 12 can be factored as 2(6) or as 3(4).
- A "prime" number is a number whose only factors are itself and 1. For example, 3, 7, and 13 are prime numbers.
- "Prime factoring of monomials" means taking an expression with no addition or subtraction in it, and rewriting it as a product of numbers that cannot be broken down any further. For example, the prime factorization of 40 is $2 \bullet 2 \bullet 2 \bullet 5$.

Rules:

- Write the number to be factored.
- If the number is negative, the first factor will by -1. Otherwise select a prime number that divides evenly into the original number. Divide, placing the factor and the answer under the number, like two branches of a tree.
- Continue selecting prime numbers and breaking down previous number into tree branches.
- List all the prime factors as a multiplication expression.
- Summarize the answer by writing the prime factors using exponents when possible.

Example:



$60 = 2 \bullet 2 \bullet 3 \bullet 5 = 2^2 \bullet 3 \bullet 5$

Practice Problems:

Use exponents to list the prime factors of each of these numbers.

96. 68

97. 88

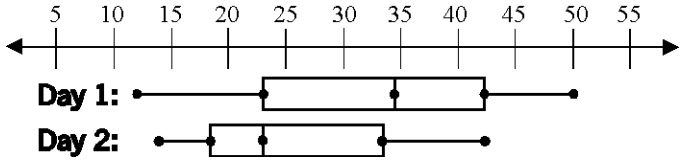
54. 108

98. What monomial has these factors: $2 \cdot 3 \cdot 4 \cdot x \cdot x \cdot x \cdot x \cdot x$?

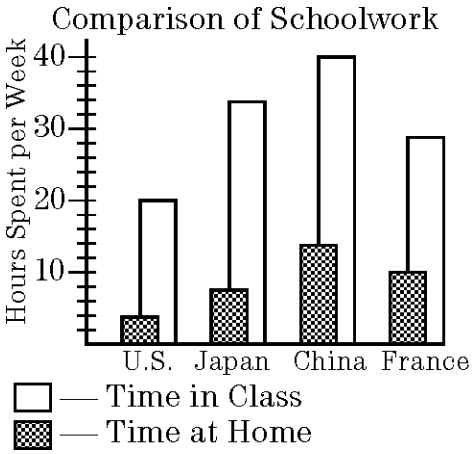
12: Reading Charts and Graphs

Answer each question below and explain your reasoning.

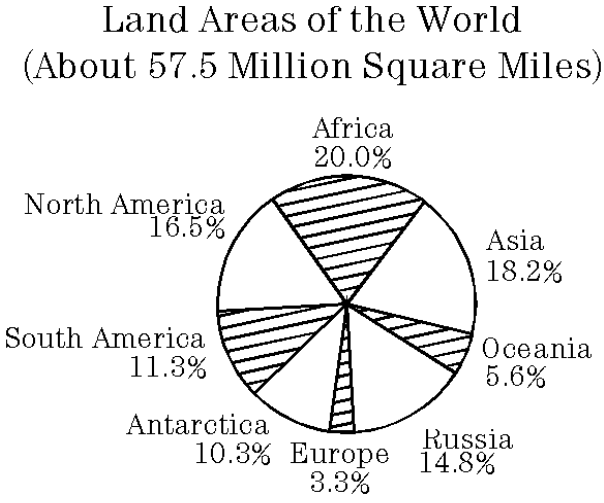
99. The box-and-whisker plot shows the results of tests given on two different days. On which day were more than 75% of the scores higher than the median of the other day's scores? Explain your choice.



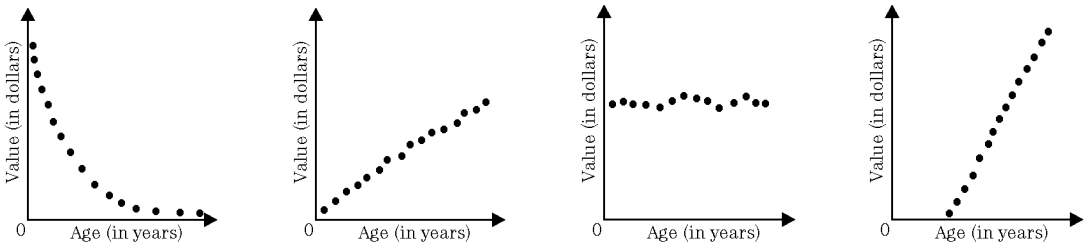
100. The graph shows a comparison of hours spent on schoolwork.
   a. How many hours do Japanese students spend on their schoolwork at home?
   b. What percentage of schoolwork time is spent by the French students in doing their in class?



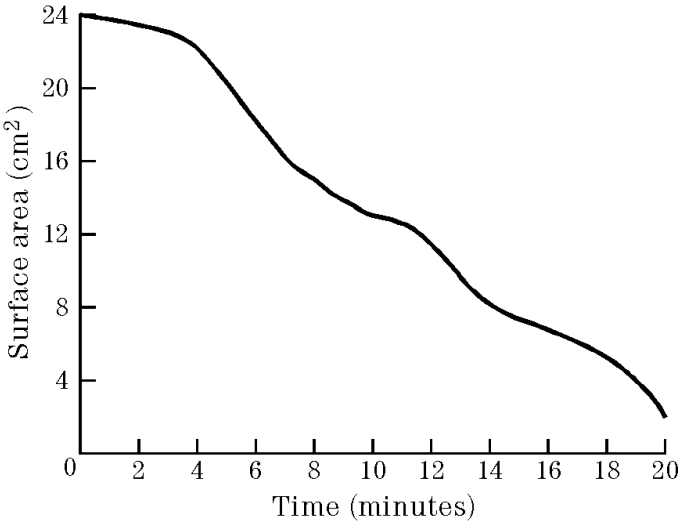
101. What percentage of the land area of the world is taken up by North and South America?



102. The computer age has advancements occurring in technology on a daily basis. Which of the following graphs would **best** represent the age of a computer compared to its value? Explain your reasoning.



103. Everyone has seen cartoons of ants carrying off entire plates of food. On a more realistic note, the graph below represents a sugar cube as it is eaten by several ants. What is the approximate surface area of the cube after the ants have been eating the cube of sugar for 8 minutes?



104. The graph shows energy consumption in the United States for the years 1960-1990. According to the graph what was the approximate increase in energy consumption between the years 1970 and 1980?